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I. Main duties of the research unit between the 1st of January and the 31st of August, 2019

The fundamental goal of the Alfréd Rényi Institute of Mathematics is to pursue research of high international standing in pure mathematics. The Institute is an important centre of mathematics internationally. Fellows of the Institute received several Hungarian and international recognitions in 2019. One researcher was elected as regular member and two as corresponding member of the Hungarian Academy of Sciences. During the reporting period five research groups in the Institute were funded by the ERC. A consortium led by the Institute continued its research activities in the mathematical foundations of artificial intelligence, funded by the National Research, Development, and Innovation Office.

Researchers of the Institute published 151 scientific papers during the reporting period. The most important results have appeared in the most significant international mathematical periodicals (Advances in Mathematics, Journal of the European Mathematical Society, Annals of Probability, Journal für die reine und angewandte Mathematik, etc.).

The scientific tasks of the Institute concentrate on fundamental research. However, significant efforts were devoted to some topics in applied mathematics as well. The main applied areas investigated in the Institute are artificial intelligence, the theory of large networks, cryptography, financial mathematics, as well as bioinformatics. Mathematical statistics has also been applied in several related areas (e.g., in astronomy and in environmental science).

The Institute is organized in the framework of 8 scientific departments, 5 Momentum research groups, 2 ERC research groups, and the research group on mathematics education. The research topics of the Institute are continuously adjusted to the most recent developments in mathematics.

II. Outstanding research and other results between the 1st of January and the 31st of August, 2019

a) Outstanding research results

Low Dimensional Topology Momentum Research Group

They developed a new knot invariant, based on the double branched cover of the knot and the covering involution on that space. They adapted certain Heegaard Floer theoretic constructions to this setting, including involutive Heegaard Floer homology. Using these new invariants, novel linear independence results in the knot concordance group have been verified.

In the topic of high dimensional contact topology, they made advances in the study of symplectic fillings of certain explicit 5-dimensional contact manifolds (stemming from the Bourgeois construction); they gave obstructions to the existence of such fillings and classified

them in some cases. They continued their search for contact manifolds for which the contactomorphism group contains large subgroups.

They proved the numerical version of the $P = W$ conjecture in the Painlevé 1-6 cases, and its geometric version in the Painlevé 6 case. They classified all the cases where the Hitchin map on a two complex dimensional moduli space of irregular Higgs bundles on the Riemann sphere has a unique singular fiber.

In a joint project they studied the connection of local and global invariants of a curve in a normal surface. From the local point of view, they have investigated the delta invariant, while from the global point of view the so-called kappa invariant came into the research, which measures the obstruction for the twisted canonical divisor to have logarithmic poles along the strict transform of the curve. They have proved the equivalence of these two invariants for any curve embedded in rational surface singularities.

They continued their study in the real Steiner problem (how many conics are tangent to five given conics in the real projective plane). They show that almost all even natural number below 3264 can be the cardinality of the solutions. This statement can be further refined using the Welschinger invariant.

Automorphic Forms Momentum Research Group

Automorphic forms are harmonic waves with a rich symmetry, which help us to understand the whole numbers. The members of the research group examined how many cusp forms can violate the Selberg conjecture on a compact arithmetic hyperbolic surface. As a by-product of this research, they proved a Minkowski type result for linearly independent subsets of ideal lattices arising from number fields. On a concrete arithmetic hyperbolic 3-manifold, they gave a good approximation for the number of prime geodesic lengths in short intervals. On the group $SL(2,\mathbb{C})$, they studied the maximum of non-spherical Maass forms. They proved optimal bounds for multidimensional exponential sums that play an important role in the distribution of arithmetic objects. They extended various formulae of Hurwitz, Waldspurger, Katok-Sarnak, Bykovskii, and they initiated the generalization of a formula of Motohashi. They showed the overconvergence of multivariable (φ,Γ)-modules, and they verified that the generalized Herr complex computes Galois cohomology in this case. Finally, they made progress in a graph theoretic question: how many Hamilton paths can be given on n vertices so that the union of any two contains a cycle of length $2k$.

Groups and Graphs Momentum Research Group

In the framework of preparing for the DYNASNET Synergy program starting September, they investigated graph theoretical questions related to control problems for Barabási type networks. Specifically, they analysed the solutions for the matching problem with randomized local algorithms.

They continued investigating the growth of mod p rank and torsion in higher homologies. They proved a general vanishing result that, among other things, can be applied to lattices in Lie groups where the homology growth has not been known. They finished their paper on the local convergence of eigenvectors of locally symmetric spaces and the paper on uniform limit multiplicities.

They worked on the expansion of vertex weighted graphs. It turned out that when weighting a graph with a random walk, the corresponding weighted graph sequence is always hyperfinite. They showed that the independent intersection of co-amenable IRSs is always co-amenable.

They managed to improve the theory of intermediate density graph convergence that came out in 2018. They proved an intermediate version of the Szemerédi regularity lemma and adapted the counting lemma to this setting. These two lemmas open the possibility that one will obtain a graph-like limit object for intermediate density graph sequences.

Large Networks Momentum Research Group

They studied the extension of higher order Fourier analysis to arbitrary compact Abelian groups. Related to holographic functions and neural networks they studied what types of classification problems can be performed efficiently with neural networks. The connection between many different notions was explored. The most important ones are: holographic property; approximability with low degree polynomials; representability with shallow neural networks. A generalization of Szemerédi's regularity lemma was successfully applied. This led to promising qualitative results.

They studied the most prominent variational autoencoder model for anomaly detection. A new method was suggested to correct weaknesses. They worked out a method using optimal transport that improves on learning efficiency if the training set is not assumed to come from independent sampling. They obtained results in automated proofs using deep learning. They studied how to generalize from short to longer proofs.

They used methods from combinatorial optimization together with continuous methods that can take the latent image of the training set close to a given prior. They continued the study of the eigenvectors of non-symmetric random matrices. They further developed methods that were earlier applied to the convergence of random graphs. In another project they gave sufficient condition for a branching Markov process to be approximable by colourings of regular graphs. Convergence speed for preferential attachment models were also investigated.

They started a larger project to study the size of the largest independent sets in random regular graphs. The famous paper by Ding and Sly answers this question when the degree is big. It is conjectured that if the degree is at least 20, then similar results apply. However, the case of the degree being at most 20 is mostly open. A new approach was found when the degree is at least 2000. They investigated the connections between various graph limit notions including *s*-convergence and fractional quotient convergence.

They studied the complex zeros of the partition function of the anti-ferromagnetic Ising model on bounded degree graphs. Using methods from complex dynamics they could more precisely measure the positions of the roots of spherically symmetric trees. They proved that these roots are significantly different from the roots of Cayley trees. These methods were extended to the hard-core model and the Potts model.

Financial Mathematics Momentum Research Group

The Financial Mathematics Momentum Research Group working in the framework of the Department of Probability Theory and Statistics continued their investigations about Markov chains in random environments. Such problems appear e.g. in queuing systems with stationary

service times and in stochastic volatility models of financial mathematics. They also studied convergence properties of important algorithms in machine learning: the Kiefer-Wolfowitz and the stochastic gradient Langevin algorithms.

Growth in Groups ERC Research Group

Generalising earlier results, they proved that there exists a compact manifold on which every finite 2-nilpotent group of bounded rank acts faithfully. They also proved an analogous statement for the birational automorphism groups of complex varieties, extending a result of Zarhin.

They also studied the following conjecture: If G is a finite p-group such that every elementary abelian normal subgroup can be generated by r elements, then every abelian subgroup A of G can be generated by $2r$ elements. They proved that in the cyclic decomposition of A there are at most $2r$ cyclic factors of order at least p^2, if p is at least 3.

They proved that for a connected Riemannian manifold of dimension at least 4, the following properties are equivalent: (1) the manifold is locally harmonic; (2) the total scalar curvature of a tube of small radius about a regular simple arc depends only on the length of the arc and the radius of the tube; (3) the total scalar curvature of a tube of small radius about a regular geodesic segment depends only on the length of the geodesic and the radius of the tube. They proved that property (2) characterizes among 3-dimensional manifolds the class of D'Atri spaces. This class is strictly larger than the class of locally harmonic spaces, and properties (2) and (3) are equivalent if the manifold has bounded sectional curvature.

The number $k(G)$ of conjugacy classes of a finite group G is equal to the number of complex irreducible characters of G. They proved that if G is a finite group and p is a prime whose square divides the order of G, then the number of conjugacy classes of G is at least cp.

Let G be a finite group and H a nilpotent subgroup of G. They proved that if a character of H induces to an irreducible character of G, then the generalized Fitting subgroup of G is nilpotent. This implies a main result of a paper of Riese and Schmid. As a by-product of these investigations, they obtain a generalization of a result of Vdovin, namely that the order of any nilpotent subgroup of any almost simple group G is less than the square-root of the order of G. They showed that if V is an n-dimensional vector space over the finite field F, and G is a coprime primitive linear group, then by randomly choosing 11 vectors from V, the intersection of the corresponding stabilisers is trivial with large probability, provided that the order of V is large enough.

Noise-sensitivity Everywhere ERC Research Group

They have determined the exact mixing time of some random walks on finite symmetric groups, generated by the transpositions given by the so-called dumbbell graphs. They have formulated a general conjecture regarding which graphs produce transposition random walks that exhibit the so-called mixing time cutoff phenomenon, a fashionable research topic in the area of Markov chains. Their method also solves a conjecture of Lacoin and Leblond (2011), giving a new, elementary proof and a generalization for the famous Diaconis-Shashahani (1981) theorem.

The group have started to develop generalizations of the classical noise-sensitivity tools and results to fractal percolation, to the Ising model of magnetization, and to Gaussian processes. They have proved for the high temperature Curie-Weiss model that the output of transitive functions cannot be guessed from a small part of the input. The proof uses information theoretic tools.

They have answered a question of Gil Kalai: there exists a sequence of monotone transitive Boolean functions that are noise stable but have pivotal bits with high probability. It follows from the construction that also volatile Boolean functions with the above properties exist.

When can unimodular random planar graphs be drawn in the plane in such a way that the (random) drawing is preserved by the isometries of the plane? Here the plane can be Euclidean or hyperbolic, and the widely studied class of unimodular random graphs is the natural generalization of the simple random rooted graph when one takes a finite graph with a uniform random root. An almost complete characterization of the possible invariant embeddings was given. In a related project it was shown that every unimodular random planar graph can be locally approximated by finite graphs.

Fractal dimensions of a generic compact set or the image of a generic continuous function have been calculated, where the notion of generic is being winning for Schmidt's game. Given any measure, its decomposition into conditional measures supported on certain random sets was studied, with main examples being Brownian paths and fractal percolation. This provides new tools in the geometric measure theory of intersections with these random sets.

Didactics Research Group

In the framework of Subject Teaching Research Program of the HAS, they studied the possibilities of talent maintenance of gifted students. They continued the experimental teaching program started in September 2017 also not for outstanding students. They participated in organizing talent scouting camps.

Department of Algebra

Given a group of automorphisms of a finite dimensional Lie algebra, the group acts via algebra automorphisms both on the symmetric tensor algebra of the Lie algebra and on the universal enveloping algebra of the Lie algebra. The elements fixed by the group action constitute a subalgebra in both of the above algebras. In the case of the first, commutative algebra, classical invariant theory provides tools to construct generators of the subalgebra of invariants. Now it was shown how one can get generators of the latter, usually non-commutative algebra of invariants from the generators of the commutative invariant algebra. The research was conducted in the framework of a Bulgarian-Hungarian mobility project of the Hungarian Academy of Sciences.

Several mathematical topics motivate the study of semi-invariants of m-tuples of n by n matrices. For example, applications in algebraic complexity theory received considerable attention recently. A minimal generating system of this algebra of semi-invariants is known only for very few values of n and m. Now for 2 by 2 matrices, a minimal generating system was found for the so far open case of a characteristic 2 base field. Moreover, the same was done working over the ring of integers instead of a field. The latter result can be considered as the optimal characteristic free description of the generators of the algebra of semi-invariants.

Concerning the isomorphism problem of Cayley graphs, they studied the case of Abelian groups of rank 2. They were able to prove the so-called CI property for certain groups of rank 2, that had not been known in the literature. The research was conducted in the framework of a Hungarian-Slovenian collaboration. The investigation of *F*-inverse monoids considered in enriched signature was continued where they are equipped (besides inversion) with an additional unary operation which assigns to each element the greatest element of its sigma-class. Affirmative answer was given to the question whether the Cayley graph of an *X*-generated group *G* can be recovered from the universal *F*-inverse monoid corresponding to *G*.

Continuing earlier investigations, very general new results have been obtained about the Morita equivalence of semigroups without identity; among others, about the preservation of the lattice and the quantale of ideals.

By computing the Bockstein Spectral Sequence, it was shown that all torsion elements in the integer coefficient cohomology groups of even real flag manifolds are of order two. They showed that the Chern-Schwartz-Macpherson class of a complexified subvariety in an algebraic conjugation space is mapped by the degree halving isomorphism to the Sullivan class of the real subvariety. Compatibility of the cohomology frame of conjugation spaces with Gysin maps for conjugation equivariant maps has been shown.

Department of Algebraic Geometry and Differential Topology

They proved a generalisation of the conjecture of Ghys to non-compact manifolds: if a topological manifold has finitely generated homology groups, then each finite subgroup of its homeomorphism group has a nilpotent subgroup of bounded index. They studied the Liebeck-Nikolov-Shalev conjecture for large sets: if *G* is a finite simple group and *S* is a subset of size at least the 0.99-th power of the size of *G*, then *G* is the product of a bounded number of conjugates of *S*.

They investigated finite subgroups of the birational automorphism groups of varieties. They managed to show that these groups contain a large nilpotent subgroup of bounded index, such that the nilpotency class can be bounded by the dimension of the variety. Later they sharpened this bound. Currently, they work on showing that the bound can be further lowered to the constant two. They write a series of article on the properties of the Abel map associated with surface singularities.

Department of Analysis

They proved the famous Fuglede conjecture for convex bodies. Since the conjecture is not true in general, it is particularly interesting that it holds for convex bodies. They came closer to a conjecture of Erdős by giving an upper bound on the density of planar sets avoiding unit distances.

They managed to substantially sharpen a famous result of P. Šemrl, which states that the 2-local automorphisms of the full operator algebra of a Hilbert space are necessarily automorphisms. They determined the isomorphisms of the positive definite cone of *C**-algebras in the conventional sense and in the sense of Kubo-Ando power means. They summarised the characterisations of Jordan *-isomorphisms of operator algebras based on their preservation properties.

They concluded their investigation on the following question: when can it be stated that two Borel measures on a locally compact Abelian (LCA) group have the property that there exists a constant C such that the integral of any positive definite and nonnegative function f with respect to one measure cannot exceed C times the integral of f with respect to the other one? They derived a general inequality between the density of translational sets forming a packing with copies of a given "master copy set" in an LCA group and its Delsarte-type extremal constant. Also, they proved that in an LCA group and for a closed set of finite measure there always exists an extremal function for the Delsarte type extremal problem in the class of functions with given compact support of the Fourier transform.

They determined the precise Markov inequality in several variables on non-convex domains in L_p norm. They made substantial progress in generalising the Marcinkiewicz-Zygmund inequalities in L_p. In connection with weighted approximations in several variables, they gave conditions under which every continuous function, vanishing at the zeros of the weight w, is the uniform limit of weighted polynomials of a given type. They studied some Jackson-Favard type weighted approximation problems in the case of the exponential weight.

The concept of exceptional orthogonal polynomials was introduced 15 years ago by physicists in the study of quantum mechanics. They gave asymptotics for the kernel function of the projection operator projecting to the space spanned by the first n polynomials. They also studied the general distribution of the zeros of exceptional orthogonal polynomials. Since the zeroes of certain polynomials of this type form a configuration with minimal energy in a given external force field, the above investigations are also motivated by theoretical physics and practical applications.

They derived large-parameter asymptotic expansions, complete with error bounds, for particular solutions of the associated Legendre equation. They proved the Borel summability of the WKB solutions, normalised at infinity, of certain Schrödinger-type differential equations near the Stokes curves emanating from a simple pole of the potential function. They gave a complete description of the Stokes phenomenon related to these solutions. They obtained information on the singularities of the Borel transforms and proved the resurgence properties of the late coefficients in a given neighbourhood of a simple pole of the potential function. They gave a uniform asymptotic smoothing of the higher-order Stokes phenomenon in the case that the singulants are equal.

Department of Discrete Mathematics

They studied search problems in a model when you can get false answer for any question but only in the case of an element contained by a set depending on the question. They also investigated how many vertices of the n-dimensional cube can a ball of radius r contain. (The full solution of the problem in 30-40 dimensions would imply important statistical applications.) Estimates were given on the number of 2-edge-colourings of graphs not containing given monochromatic stars.

They determined the Turán number of Berge trees. They found hypergraphs with Turán number of order of magnitude not equal to any power of n. They determined the Turán number of a large class of ordered graphs and hypergraphs. They also determined the generalized Turán

number of short paths and cycles in planar graphs. The maximum number of edges of graphs with edge ordering not containing some edge ordered subgraphs was also determined. They determined a new version of Ramsey and Turán numbers when the vertex degrees of the subgraphs are restricted and proved many exact and asymptotic results. They generalized the result on 2-colour Ramsey numbers of linear trees to arbitrary number of colours.

They investigated the connected colour avoiding connected components of randomly edge coloured Erdős-Rényi graphs. It was proved that there is a unique colour avoiding giant component in case of appropriately chosen parameters. They estimated the maximal length of Armstrong codes by means of a new type of use of the linear programming bound of Delsarte. Generalizing the results of Erdős and Fowler, they showed that in case of $r = 3,4,5,6$ colours there is no monochromatic, 2 diameter component of size $n/(r-1)$.

They solved the Figlede conjecture in cyclic groups whose order is the product of four primes. They managed to prove that the swap Markov chain is mixing fast in the classes of so-called *P*-stable degree sequences. They proved the dimension free version of the classical Caratheodory theorem and its extensions. There is a similar extension of the Helly theorem. It was proved that any reasonable property or parameter can be tested in property *A* classes with an appropriate oracle. This was the first application of the property *A* class in computer science.

Department of Geometry

Improving their earlier results, they gave the strongest bound on the upper density of planar sets that do not contain two points at unit distance. They further investigated the generalizations of the Crossing Lemma for multigraphs and gave new bounds, especially in the case when the number of edges is very large compared to the number of vertices. A graph is c-crossing critical if its crossing number is at least c, but if we remove any edge, its crossing number drops below c. Improving their earlier results, they proved that the crossing number of a c-crossing critical graph is at most $2c$. The Crossing Lemma has been generalized to many different versions of the crossing number, but in most cases the improvements work only for the classical crossing number. They managed to get an improvement for several versions of the crossing number.

They studied coverings of the plane with unit discs or unit squares and gave an upper bound on the shortest path between two points, with the property that all points of the path are covered at least twice. They solved a closely related problem, where we have to bound the length of the shortest chain connecting two points, as a function of their distance. They managed to solve a famous problem of Erdős and Schur for semialgebraic colourings. Essentially, they proved that if we colour the edges of a graph of exponential size with a fixed, finite number of colours, such that each colour class has bounded semialgebraic complexity, then there is a monochromatic triangle.

They proved that if the disjointness graph of n planar, x-monotone curves contains no clique of size k, then its chromatic number is $O(k^4)$ and this bound is sharp. If we do not assume that the curves are x-monotone, then the chromatic number can be arbitrarily large. Related to this, they improved an old result of Bollobás: they constructed partially ordered sets whose Hasse diagram has chromatic number at least $c \log n$.

They proved inverse Blaschke-Sanataló type inequalities in the plane, for locally convex curves that can go around the origin several times. They proved a reversed version of the classical

Minkowski inequality, moreover, they gave an estimate on the stability of the obtained inequality.

Department of Set Theory, Logic and Topology

They continued the study of minimum possible weights of nowhere constant continuous images. They proved that for a normal source space X this is equal to the so-called strong open splitting number of X. These investigations lead to interesting Boolean algebraic questions. Surprisingly the existence of a measurable cardinal is equiconsistent to the existence of a Boolean algebra B whose shattering number is not countable, but whose shattering number is smaller than its splitting number.

They investigated the existence of kappa-homogeneous, but not kappa-transitive permutation groups on large sets. They showed that there is an omega-homogeneous but not omega-transitive permutation group on a cardinal lambda, assuming that there are only finite many infinite cardinals below lambda, or $V = L$.

They have calculated the so-called cardinal invariants of the Haar null sets of Christensen and the Haar meagre sets of Darji. They have answered a question of Mycielski concerning the length of the random interval-homeomorphism, and also proved far-reaching generalisations which are related to singular functions.

They began to investigate the algebra of concepts of special relativistic spacetime as first-order logical theory. The algebras of zero-ary and unary concepts are trivial, two-element ones. The algebra of binary concepts has 16 elements, and it has no subalgebra. The algebra of ternary concepts is infinite and it has proper subalgebras. These results are true over arbitrary ordered fields.

They elaborated a method for constructing finite "homogeneously amalgamating" structures from given finite sets of finite structures. As an application, they gave a new proof for Ehud Hrushovski's partial isomorphisms extending theorem, and showed that the cardinality of the constructed model is much smaller than the known double-exponential upper bound. Further applications are new, simpler proofs for the finite model property of the guarded fragment of first-order logic, for the finite-algebra and finite base properties of cylindric-relativized set algebras.

An element g of the automorphism group of an (infinite) first order structure is weakly generic if its conjugacy class is dense, and the g-orbit of each element is finite. They showed that if a "simple" structure has a sequence of weakly generic automorphisms of length 2 (and satisfies a further, mild technical condition), then it has tuples of generic automorphisms of arbitrary finite length.

Department of Number Theory

They proved a new inverse theorem related to the Erdős-Heilbronn problem (which is well-known in additive combinatorics) involving 3-term sums, and the possible further extension of the multilinear method was also investigated.

Earlier they studied the Prime Geodesic Theorem for the full modular group. This year the analogous higher-dimensional problem was investigated for the Picard group (this is the group

of two by two matrices with determinant one over the Gaussian integers). The short interval theorem of Bykovskii was generalized for this case. Their theorem gives an asymptotic result for the number of conjugacy classes with norm between X and $X + Y$, where Y is a power of X with exponent smaller than one. The surprising fact was observed that this problem was closely related to the Gauss Circle Problem. There are two interesting aspects of this work. On the one hand, they connected the problem to the Gauss Circle Problem, on the other hand they managed to handle surprisingly short intervals, due to the strong error term in the circle problem. As an application of the short interval theorem, they proved an error term for the Prime Geodesic Theorem for the Picard Group which is better than the previously known estimate.

They investigated approximations of the Goldbach and Twin Prime Problems with special attention to the structure of the set of possibly existing exceptional Goldbach numbers and for the frequency of such numbers.

They studied random walks on compact groups. A necessary and sufficient condition was given for a random walk to equidistribute in any given Borel set with probability 1. The precise asymptotics and limit distribution of the remainder term were also found. Regarding equidistribution in intervals of random walks on the circle group a functional central limit theorem and a functional law of the iterated logarithm were proved. A conjecture was made about the tightness of the conditions of the theorems.

Department of Probability Theory and Statistics

The researchers of the department greatly generalized results about consensus algorithms on networks with asynchronous communication loss, applying them to random matrix products, also identifying the speed of convergence. They have demonstrated the significant efficiency gain by only a single new edge in the mixing problem of Markov chains. They have investigated parametric Poisson equations, found at the heart of stochastic approximation algorithms, showing favourable properties of the solutions.

They proved an invariance principle for particle trajectories in the random Lorentz gas and extended it to Eherenfest's so-called Wind-Tree model. Both are notoriously difficult models in non-equilibrium statistical physics and this is the first major progress in forty years. They analysed random permutation processes and proved that the so-called random stirring process on $\mathbb{Z}^d$ in equilibrium asymptotically loses its geometric structure. Analysis of random stirrings on $\mathbb{Z}^d$ is motivated by fundamental questions of quantum statistical physics.

The properties of comb-type random walk on two-dimensional space with integer coordinates were investigated. Weak and strong invariances were proved for such comb-type random walks, extending the results concerning random walks on combs. They investigated statistical properties of random permutations. They studied the structure of sumsets (Freiman type problems) using probabilistic tools. They showed that any two d-edge-colourings of any d-regular bipartite graph can be transformed into each other via d-edge-colourings such that in each step the perturbed edges have at most 3 colours. These transformations might be a kernel of a Markov chain Monte Carlo algorithm such that the inverse of the Metropolis-Hastings ratio has a polynomial upper bound. They showed that the Pareto front of the Minkowski product is Pareto front of the Minkowski product of the Pareto fronts.

Applied research

The research carried out at the Rényi Institute has focused on exploratory (theoretical) problems. On the applied research projects the research groups of the previous years have continued their work in financial mathematics, cryptography, bioinformatics and other mathematical methods applied in life sciences, e.g., in neural networks.

The Deep Learning Research Group, operating partly in the framework of the *Large Networks Momentum/ERC Research Group* – beyond the theoretical results detailed in the report of the research group – has worked on the following themes.

Exploiting the successes of deep reinforcement learning in the area of automated theorem proving is a promising new research programme. The Deep Learning Research Group presented an automated theorem prover guided by a deep learning model that is capable of generalizing from short arithmetic proofs to longer proofs with a similar structure. The synthetic datasets created to evaluate the system were published as a benchmark for similar future systems.

Another main research interest was the deeper understanding of autoencoder-based generative deep leaning models. The group has created a method to alleviate the notoriously bad out of sample likelihood estimates made by variational autoencoders. A work in progress is to create a Wasserstein autoencoder based model that is capable of optimizing a divergence between the latent image of the full dataset and a given prior.

The team participated in the organization of several workshops, with the main goal of knowledge sharing within the Hungarian AI community.

The *Cryptology Research Group* of the Rényi Institute mainly focused on further problems and generalizations of previous results on secret sharing in the first eight months of 2019. They have significant result in a special case of graph based secret sharing schemes: they determined the exact information ratios of unicyclic graphs. The results are not restricted to the worst-case scenario but can be applied partially to the average case as well. On the other hand, they examined the duality problems on access structures describing secret sharing. They developed novel methods related to matroid theory which are of independent interests, and in the linear case they got the best possible results. They have participated on several international cryptology conferences, and a publication has been accepted.

The researchers of the *Bioinformatics Research Group* of the Rényi Institute conducted collaborations with UNSW Sydney and the Eötvös Loránd University. The focus of their research was studying mixing time properties of Markov chains on realizations of degree sequences and degree-constrained edge packing problems. The former project is needed to generate null hypotheses in statistical inferring of networks. The latter project has many applications in discrete tomography. The results have been published in journals Informatica and Acta Math. Univ. Comenianae, and was also presented at the EUROCOMB 2019 conference. They also published a book titled “Computational Complexity of Counting and Sampling” in which there is a special emphasis on bioinformatics applications like sequence alignment, structure prediction of biological macromolecules, genome rearrangement and network algorithms.

The researchers of the Financial Mathematics Momentum Research Group continued their investigations about Markov chains in random environments. Such problems appear e.g. in queuing systems with stationary service times and in stochastic volatility models of financial mathematics. They also studied convergence properties of important algorithms in machine learning: the Kiefer-Wolfowitz and the stochastic gradient Langevin algorithms.

Career advancement of researchers

One researcher of the Institute has been elected as regular member of the Hungarian Academy of Sciences, while two further members have been elected as corresponding members, and six young researchers received their PhD degrees. At the end of the reporting period 9 members of the Academy, 28 doctors of HAS, 65 researchers with PhD or CSc worked at the Institute; 19 researchers have not yet obtained a degree. Besides the regular employees 20 emeritus research professors (9 members of the Academy, 11 with DSc title) take part in the scientific work of the Institute. The Institute puts great emphasis on involving young talents – working towards their PhD or just obtaining the degree – into the research work. In the reporting period 13 young researchers worked in the Institute on positions provided by the Hungarian Academy of Sciences in the framework of its Young Researcher Program. The Institute has an agreement with the Central European University (CEU). In this framework many doctoral students were supervised by members of the Institute. This contract expired in August 2019, and since CEU moved to Vienna, it has not been renewed.

b) Science and society

Most of the research topics in pure mathematics are not suitable for discussions for the general public. On the other hand, the international success of the researchers has underlined the importance of the research conducted in the Institute even in public media.

The researchers of the Institute play an important role in popularizing mathematics, giving lectures for high school and university students. The Institute regularly organizes an open house during the Festival of Hungarian Science, where high school students and their teachers can get information about the mathematics profession. In April they have organized the Day of Girls, when over forty young female high school students visited the Institute and got a glimpse into the work carried out here and into some interesting mathematical ideas.

Members of the Institute take part in fostering mathematical talents. In 2019 they have organized several mathematical camps and other events for students interested in the subject. The Institute plays a role in giving scientific background for the teachers of specialized mathematics classes in high schools.

III. A presentation of national and international R&D relations between the 1st of January and the 31st of August, 2019

National relations

Researchers of the Institute teach part time at many universities both in Budapest and in other cities (Eötvös University, Budapest University of Technology, Budapest Business School, National University of Public Service, Péter Pázmány Catholic University, University of Szeged, Pannon University). They play an important role in doctoral schools and in Masters programs. 13 fellows of the Institute are core members of doctoral schools in various

universities, and 27 of their researchers supervise 52 doctoral students. Especially important was the collaboration between the Institute and the Department of Mathematics and its Applications of the Central European University. The lecturers and the supervisors of the Masters and doctoral programs of CEU mainly belonged to the Institute, including the department chair and the leader of the doctoral program. Also, a large number of lecturers of the Budapest Semesters in Mathematics English language study abroad program for North American students belong to the Institute. This program helps to bring the fame of Hungarian mathematics to American universities, and serves as a role model for other international programs.

For the Institute the close contact with the new generation of mathematicians is of utmost importance. In this spirit 60 members of the Institute (50 percent of all researchers) were active in teaching at universities; it included supervising 11 student research projects, 21 BSc and 16 MSc theses.

As part of the renewal program of the Academy, the Institute restarted its guest researcher program, which enables university professors and lecturers to spend one or two semesters in the Institute freed from their teaching duties. As part of this program, during the reporting period one guest from the Budapest University of Technology, two from Eötvös University and three from the University of Szeged joined the research teams of the Institute (spending here 39 months altogether).

The weekly seminars in the Institute are attended regularly by researchers from other institutions, among them some people from universities outside Budapest. In this way these seminars influence the entire mathematical scene in Hungary.

Members of the Rényi Institute traditionally take part in various Hungarian scientific committees well over proportion. In particular, the Section of Mathematics of the Hungarian Academy of Sciences (HAS) and its committees, the Hungarian Research Fund (NKFIH), and the János Bolyai Mathematical Society (BJMT) can be mentioned. One of the vice-chairmen of the Bioinformatics Committee of the HAS, the president and the deputy secretary general of the BJMT, the chairman of the Scientific Section of the BJMT, and the vice-chairman of the Applied Mathematics Section of the BJMT are all researchers of the MTA Rényi Institute.

International relations

The researchers of the Institute have very extensive international relations. Among the co-authors of the members of the Institute one finds mainly mathematicians from outside Hungary. Joint projects and jointly organized conferences are also typical.

In the reporting period 34 members of the Institute were involved in organizing international conferences, some of them even on several occasions. In the Summer of 2019 the Institute organized four major conferences and a smaller scale workshop.

The visits in the framework of the bilateral exchange programs between the Hungarian Academy of Sciences and its partner institutions successfully contributed to the cooperation with foreign partners. With the help of these programs fruitful joint research projects, useful exchange of information, and conference participations were made possible.

Researchers of the Institute took part in 13 international scientific committees altogether. Names of the Institute's researchers can be found 195 times on lists of editorial boards of various international journals. In the reporting period the researchers gave altogether 272 talks at international meetings, and many of these were given as invited or plenary lectures.

In the reporting period 6 researchers spent more than half a year abroad at the following institutions: University of Chicago (USA), City University of New York (USA), Lancaster University (England), Universität Hamburg (Germany), and Università di Pisa (Italy).

Financed by the ERC and Momentum grants or from other sources, 18 foreign researchers worked in the Institute for a longer period, and a further 5 foreign researchers spent 1-8 months (altogether 23,5 months) in the Institute from the USA, China, Sweden, France and Spain. The number of foreign visitors of the Institute during the reporting period – not counting the conference participants – was 66 researchers.

IV. Brief summary of national and international research proposals, winning between the 1st of January and the 31st of August, 2019

National grants

The Rényi Institute, similar to the practices of the previous years, successfully participated in the national NKFIH researcher-initiated project proposals. In 2019 four further research projects and one young research fellow project won support together with one NKFIH postdoc research project. (This year the calls for projects and therefore the results were published later than the usual timeline and the recent report is due earlier about the first 8 months of the year, by the time of submission only the results are known but the contracts have not been signed yet.)

As in the previous years, the financial impact of these projects has varied among the incomes of the Institute. The income on these projects in 2019 is much below the level of previous years, even if count proportionally for 2/3 of the year. The variance is not due to the average of the yearly gained support, but rather due to the increasing delay in the acceptance of the research reports as well as in the prefinancing of the running projects. At the time of compilation of the recent report the institute advances the costs of NKFIH researcher-initiated projects, despite the fact that these projects are run on a prefinancing scheme. Still, due to prefinancing of other research costs and the continuity of the winning projects, this variance poses no problem in continuing the financed research.

In the project calls of the Hungarian Academy of Sciences one young researcher received a Premium Post Doctorate Research grant and – due to the high number of ERC projects – the Institute was very successful in the EUHUNKPT project call.

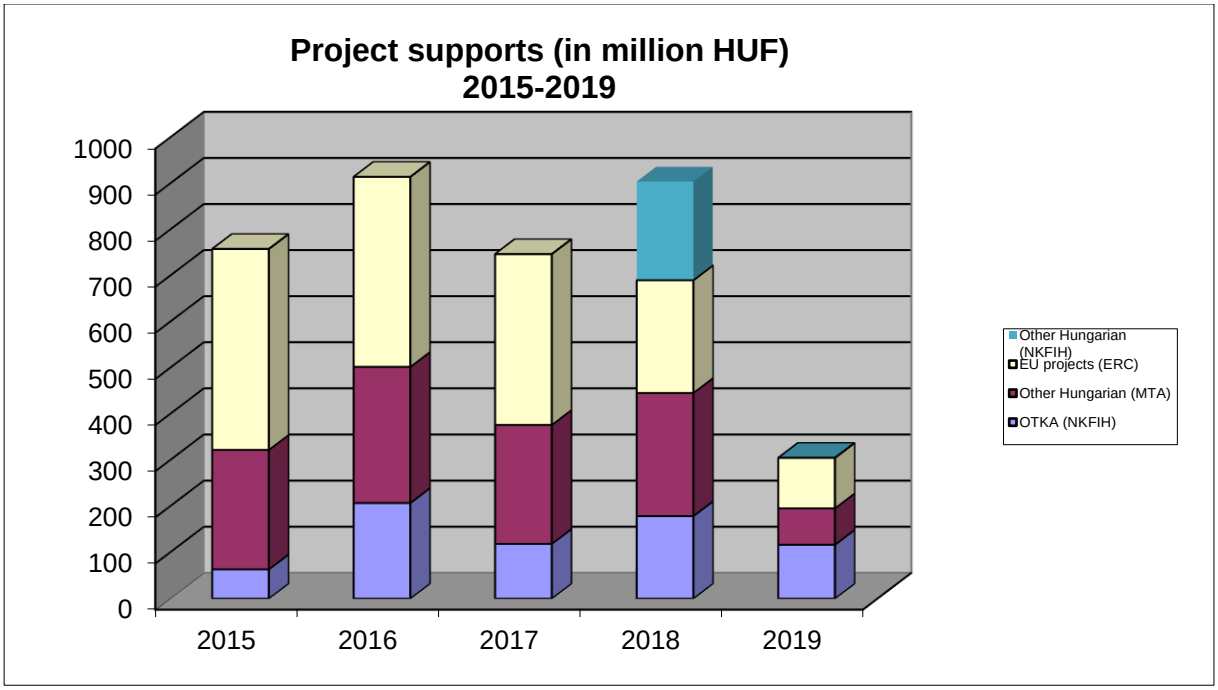
International grants

The most promising and successful international calls for the explanatory (theoretical) research projects of the Rényi Institute are EU European Research Council (ERC) calls and the mobility (Marie Sklodowska Curie) calls of the European Union. None of the project proposals submitted in 2018 was supported but one ERC Advanced Grant proposal proceeded to level 2, got "A" level evaluation, worth of supporting (and so eligible to submit a project proposal to the NKFIH Frontier – Research Excellence Programme call).

In 2019 two further ERC Advanced grant, two ERC Starting grant, and five MSC-IF grant applications were submitted, none of which has been finally evaluated yet.

Altogether the total research grant income of the Institute in 2019, counted proportionally for 2/3 of the year, is much below the level of previous years, mainly due increasing delay in financing the to the new Hungarian research and development (National Excellence Programme) grant projects, completed ERC projects (with the last stage of financing held back) and the lower cost of the initial year of the new, just starting ERC projects.

The following diagram shows the amount of project support received during the last 5 years.



V. List of important publications between the 1st of January and the 31st of August, 2019

1. Pach J, Rubin N, Tardos G: Planar point sets determine many pairwise crossing segments, In: Charikar M, Cohen E (szerk.) STOC 2019 Proceedings of the 51st Annual ACM SIGACT Symposium on Theory of Computing New York (NY), Amerikai Egyesült Államok: Association for Computing Machinery (ACM), 1158-1166. (2019)
http://real.mtak.hu/101917/

2. Böröczky KJ, Ludwig M: Minkowski valuations on lattice polytopes, JOURNAL OF THE EUROPEAN MATHEMATICAL SOCIETY 21:(1) 163-197. (2019)
http://real.mtak.hu/60186/

3. Titkos T: Arlinskii's iteration and its applications, PROCEEDINGS OF THE EDINBURGH MATHEMATICAL SOCIETY 62:(1) 125-133. (2019)
http://real.mtak.hu/60444/

4. Rössler D, Szamuely T: Cohomology and torsion cycles over the maximal cyclotomic extension, JOURNAL FUR DIE REINE UND ANGEWANDTE MATHEMATIK 752: 211-227. (2019)
http://real.mtak.hu/73294/

5. Berkes I, Borda B: On the law of the iterated logarithm for random exponential sums, TRANSACTIONS OF THE AMERICAN MATHEMATICAL SOCIETY 371: 5 pp. 3259-3280., 22 p. (2019)
http://real.mtak.hu/83658/

6. Blomer V, Harcos G, Maga P: On the global sup-norm of GL(3) cusp forms, ISRAEL JOURNAL OF MATHEMATICS 229:(1) 357-379. (2019)
http://real.mtak.hu/85308/

7. Halasi Z, Liebeck MW, Maróti A: Base sizes of primitive groups: Bounds with explicit constants, JOURNAL OF ALGEBRA 521: 16-43. (2019)
http://real.mtak.hu/89801/

8. Roche-Newton O, Ruzsa IZ, Shen C-Y, Shkredov ID: On the size of the set AA+A, JOURNAL OF THE LONDON MATHEMATICAL SOCIETY 99:(2) 474-494. (2019)
http://real.mtak.hu/103296/

9. Darji UB, Elekes M, Kalina K, Kiss V, Vidnyánszky Z: The structure of random automorphisms of countable structures, TRANSACTIONS OF THE AMERICAN MATHEMATICAL SOCIETY 371:(12) 8829-8848. (2019)
http://real.mtak.hu/103298/

10. Bianchi G, Böröczky KJ, Colesanti A, Yang D: The L_p-Minkowski problem for $-n < p < 1$, ADVANCES IN MATHEMATICS 341: 493-535. (2019)
http://real.mtak.hu/89748/

11. Halasi Z, Maróti A, Pyber L, Youming Q: An improved diameter bound for finite simple groups of Lie type, BULLETIN OF THE LONDON MATHEMATICAL SOCIETY 51:(4) pp. 645-657. (2019)
http://real.mtak.hu/92174/

12. Backhausz A, Szegedy B: On the almost eigenvectors of random regular graphs, ANNALS OF PROBABILITY 47:(3) 1677-1725. (2019)
http://real.mtak.hu/103299/

13. Aceto P, Alfieri A: On sums of torus knots concordant to alternating knots, BULLETIN OF THE LONDON MATHEMATICAL SOCIETY 51:(2) 327-343. (2019)
http://real.mtak.hu/98398/

14. Juhász P: Talent Nurturing in Hungary: The Pósa Weekend Camps, NOTICES OF THE AMERICAN MATHEMATICAL SOCIETY 66:(6) 898-900. (2019)
http://real.mtak.hu/101971/

15. Farkas Á: Dimension approximation of attractors of graph directed IFSs by self-similar sets, MATHEMATICAL PROCEEDINGS OF THE CAMBRIDGE PHILOSOPHICAL SOCIETY 167:(1) 193-207. (2019)
http://real.mtak.hu/103300/